Using curvature to infer COVID-19 fractal epidemic network fragility and systemic risk (Last Updated: 27-Mar-2020)

Danillo Barros de Souza
Departamento de Matemática, Universidade Federal de Pernambuco (UFPE), Recife, 50670-901, Brazil. *

Fernando A. N. Santos
Departamento de Matemática, Universidade Federal de Pernambuco (UFPE), Recife, 50670-901, Brazil. * *and Department of Anatomy & Neurosciences, Amsterdam UMC, Vrije Universiteit Amsterdam, Amsterdam Neuroscience, 1081 HZ, Amsterdam, The Netherlands*†

Everlon Figuerôa dos Santos, Jailson B. Correia, Hernande P. da Silva, José Luiz de Lima Filho, Jones Albuquerque
Instituto para Redução de Risicos e Disastres de de Pernambuco - Universidade Federal Rural de Pernambuco
Laboratório de Imunopatologia Keizo Asami (LIKA) - Universidade de Federal de Pernambuco (UFPE)
Recife, 50670-901, Brazil.
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The damage of the novel Coronavirus disease (COVID-19) is reaching unprecedented scales. There are numerous classical epidemiology models trying to quantify epidemiology metrics, which usually lead to exponential growth. However, it was recently observed [1] that under a complex systems perspective the epidemic outbreak in China may obey a fractal or small world kinetics as predicted by [2]. This would lead to a reliable empirical estimate for the growth of the pandemic that deviates from the standard exponential growth. In this paper, we join the ideas developed above with a recent metric developed to infer network fragility and systemic risk, the disrete Ricci curvature of the network. We assume that the growth of the epidemic in different places around the globe is also fractal, and we add noise and delays in relation to the starting of the pandemics. Under these assumptions, we are able to simulate a worldwide dynamic epidemic network. Also, using the Forman-Ricci curvature, we can estimate the fragility and risk of the network at each stage of the simulated pandemic. Lastly, we compare our simulated results with real epidemic data available from the World Health Organization (WHO). This allows to detect early warning signs that might resemble the emergence of the pandemic. The strategy above, together with other tools for spreading and modeling network dynamics, can be readily implemented on a daily basis as tools to quickly estimate the growth, risk and fragility of real COVID-19 fractal epidemic networks at different scales.

I. INTRODUCTION

Epidemic outbreaks represent a significant concern for the global health. Currently, the COVID-19 outbreak has caught the attention of researchers worldwide due to its rapid spread, high fluctuation in the incubation time and uncertain health and economic outcomes. One of the most urgent challenges of this outbreak concerns the implementation of a coordinated and continuous data driven feedback system that could quantify the spread and the risk of the epidemic, even when data is heterogeneous and subject to noise. This would allow to develop adequate responses at different scales (global, national or local) and allocate limited resources in the most effective ways.

Recent developments in topological and geometric data analysis [3–7] offer useful perspectives regarding real data treatment and has yielded outstanding results over the past years across many fields [8–11]. As an emerging and promising approach in network science and complex systems more generally [12], topological and geometric data analysis describes the shape of the data by associating high dimensional objects [3, 10, 13]. As we will illustrate with our model here, the Forman-Ricci curvature proves to be robust against white noise and, therefore, emerges as a reliable metric for monitoring the COVID-19 epidemic based on data sources that might be inaccurate due to the intrinsic nature of the epidemic spread.

Among the numerous successful interdisciplinary applications of applied geometry and topology, ranging from differentiating cancer networks [14] to modeling phase transitions in brain networks [15], one idea in particular can be beneficial to measure the systemic risk and fragility of COVID-19 epidemic networks in a data driven way: Using network curvature to infer the network fragility and systemic risk. Recently, [16] showed that it was possible to relate financial network fragility with the Oliver-Ricci curvature of a network, which emerged as a "crash hallmark" for major changes in stock markets over the past 15 years.

In this framework, the study of market fragility used these geometric tools to analyse and characterize the interaction between the economic agents (the nodes of a financial network) and its correlation levels (which defines the edges weights). In addition, these tools also allowed them to track the curvature of the financial network as a

* danillo.dbs16@gmail.com
† f.nobregasantos@amsterdamumc.nl

function of time, i.e. how the shape of the financial network changed according to a dynamic economic scenario.

As a result, the Oliver-Ricci curvature emerged as a strong quantitative indicator of the systemic risk in financial networks. From a implementation perspective, proved that an alternative, simpler discretization for computing the Ricci curvature, known as the Forman-Ricci curvature [17], is analogous to the Oliver-Ricci curvature, with the added value that the Forman-Ricci curvature has a faster computation time in large-scale, real-world networks. Therefore, this paper will use the Forman-Ricci curvature as an estimator of fragility in an epidemic network, defined as follows[18]:

$$\begin{aligned} \mathrm{F}(e) =& \#\{triangles\ containing\ e\} + 2 \\ & - \#\{edges\ parallel\ \ to\ e\}, \end{aligned} \tag{1}$$

where a *parallel edges* to e are the edges that are sharing a node or a triangle with e, but not both.

Based on a similar reasoning, and given that both epidemic and financial networks are built on correlations between time series, we use analogous geometric tools to provide a novel application of Forman-Ricci curvature to infering the fragility and systemic risk of epidemic networks, in particular, the COVID-19 network.

In a nutshell, the first step is to create an epidemic network, consisting of edges and links, based on the reported epidemic time-series. We define each spatial domain of the epidemic as the node of a network, and the links between two locations are given by the Pearson correlation coefficient (or any similarity measure) between their epidemic time-series. A simple way to access the number of cases in an epidemic network is to use the fractal growth hypothesis, as observed in [1], where the daily number of cases $n(t)$ in an epidemic follows a power-law distribution with an exponential cutoff:

$$n(t) = Kt^x \exp(-t/t_0), \tag{2}$$

where, K, x and t_0 are fitting parameters. In figure 1, we show that there is a reasonable fit between (2) and the number of reported COVID-19 cases for six countries, namely China, Italy, Iran, Spain, South Korea and the United States. This fit suggests that (2) paves a simple way for building epidemic time series that capture real-world data.

Inspired by this equation, we suggest a phenomenological model for generating epidemic time-series which can capture the growth of an epidemic network. We assume that, in each node i of the network, the daily number of cases follows a fractal epidemic growth with Gaussian noise $w_i(t)$ and a time delay d_i in relation to the epicenter:

$$n_i(t) = \begin{cases} w_i(t) & \text{if}\, t \leq d_i \\ K_i(t-d_i)_i^x \exp\left(-\frac{(t-d_i)}{t_0^i}\right) + w_i(t) & \text{if}\, t > d_i \end{cases}. \tag{3}$$

Before moving to the analysis of COVID-19 data, we show that the Forman-Ricci curvature suffices to detect fragility and risk of a simulated epidemic network. The starting point for creating a fractal epidemic network is based on simulating epidemic time series with delays from (3). In a second step, we define the weights of the epidemic network through the Pearson correlation coefficient between time series $n_i(t)$ and $n_j(t)$. The temporal epidemic network is computed for a given time window, and the process is repeated for the next time window, thus obtaining an evolving network. This approach is inspired by network analysis in other fields, such as neuroscience [19] or finance [20].

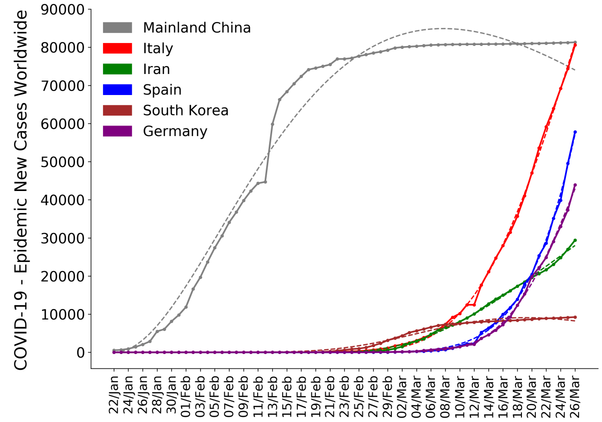

FIG. 1. COVID-19 per country. Illustration of the number of cases and fitting through fractal growth, 3, for a representative number of countries.

We illustrate the delayed epidemic time series, its Pearson correlation matrix and its corresponding network for a given time point in figure 2, thus resulting in a time evolving network.

The idea is to infer the fragility of the time evolving epidemic network by tracking geometric changes in this network as a function of time. More specifically, we observe the mean changes in the discrete version of the Forman-Ricci curvature [21] for a selected moving window for each location affected by the epidemic and use the network curvature as a indicator for its fragility and risk. Thus, we assume that the application to epidemic time-series follows an analogous behaviour to the one observed for stock markets in [16].

II. FORMAN-RICCI CURVATURE TO EPIDEMIC

As a proof of concept, we investigate a simulated time series with delays in (3). We generated 50 time series with parameters K_i, x_i, d_i, and t_0^i randomly chosen in the interval $K_i \in (0, 20)$, $x_i \in (0, 5)$, $d_i \in (10, 21)$, and $t_0^i \in (0, 1)$. We also included a small white noise with zero mean and variance of $\sigma = 0.01$. As illustrated in figure 2, the epidemic curve generated from (3) is compatible

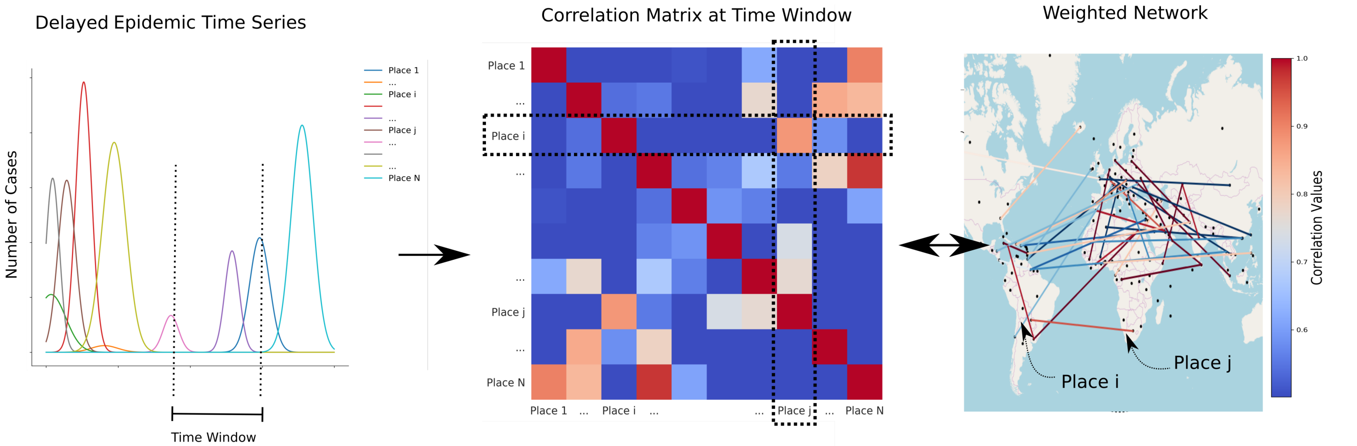

FIG. 2. Illustration of the creation of epidemic networks based on the correlations between epidemic time series across spatial domains for a given time window. This approach allow us to infer network signatures for epidemic outbreaks without relying on parameter estimation of classic stochastic epidemic approaches.

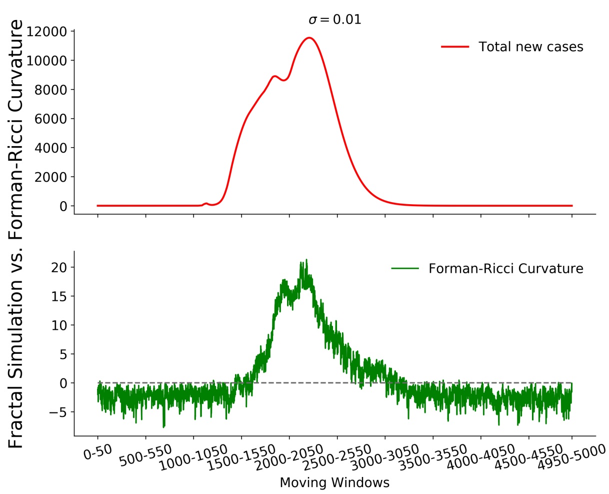

FIG. 3. (Top) Illustration of simulated epidemic curve, according to (3) and its respective Forman-Ricci curvature at the bottom, with white noise parameter $\sigma = 0.01$.

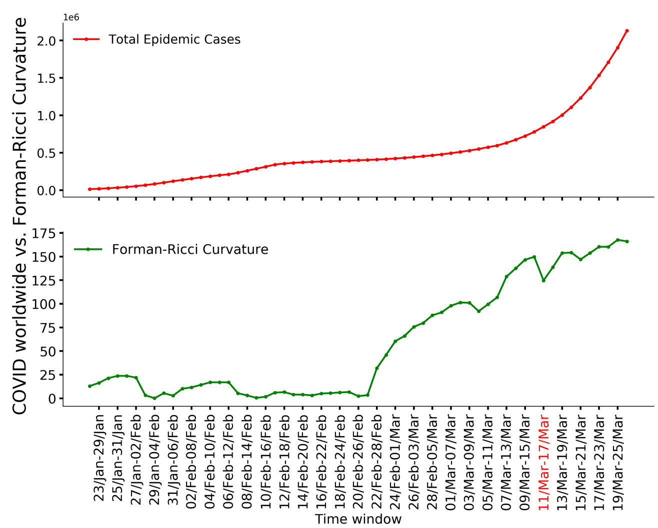

FIG. 4. Comparison between weekly world-wide epidemic curve of COVID-19 and the Forman-Ricci curvature (bottom) for the same time period. In red, we indicate the moment in which WHO declared COVID-19 as a pandemic.

with an outbreak. We contrast the simulated epidemic curve with its Forman-Ricci curvature in figure 3. We observe that the curvature is stable before the starting of the simulated outbreak and grows during the progression of the epidemic, reaching its maximum during the peak of the outbreak. After the end of the outbreak, the curvature comes back to its initial level. We emphasize that the inclusion of white noise $w_i(t)$ in our model was very important to destroy spurious deterministic correlations that appear at the end of the outbreak.

We are now ready to test whether the Forman-Ricci curvature is a reliable network fragility metric for real COVID-19 data available from the World Heath Organization. In figure 4 we illustrate both the epidemic curve (top) and the Forman-Ricci curvature (bottom) for the COVID-19 database [22]. As in the simulated data, the curvature grows with time, signaling the risk and fragility of the epidemic network. Remarkably, we observe that the curvature of the epidemic network gives an early warning sign for the emergence of the pandemics, as the curvature starts to increase way before the exponential growth in number of cases is observed.

III. CONCLUSIONS

We conclude that the Forman-Ricci curvature metric used in this paper might be a strong indicator for the fragility and systemic risk in the COVID-19 epidemic and, consequently a data driven approach to epidemic

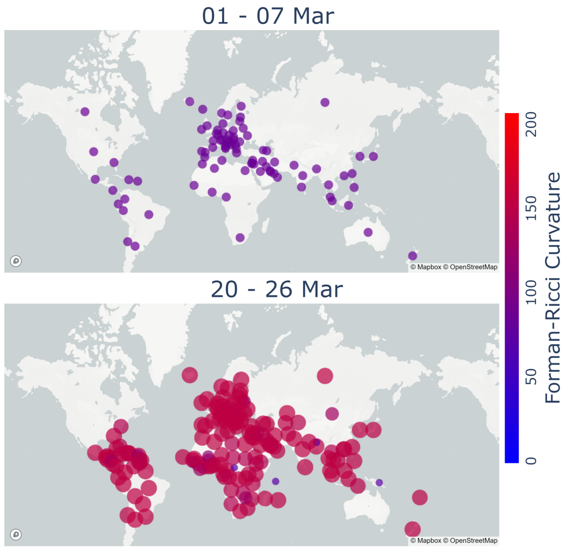

FIG. 5. World configuration based on Forman-Ricci curvature for the last 7 days of the epidemics.

outbreaks more generally. Another added value of this geometric approach, in contrast to the classical stochastics and modelling simulations, is that the results emerge intrinsically and empirically independent of parameter estimations for the pandemic, e. g. contagion rate or the basic reproduction number. This paves the way for predicting and tracking the risk of the epidemic in the absence of reliable parameter estimations. More generally, geometric and topological methods seem to emerge as promising support tools for future epidemic control policies.

IV. ACKNOWLEDGMENTS

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